IYGB – MP1 PAPER K – QUESTION 1

REWRITE IN INDEX NOTATION

$$y = \frac{8}{x} + 3\sqrt{x}$$

$$y = 8x^{-1} + 3x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -8x^{-2} + \frac{3}{2}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -\frac{8}{x^2} + \frac{3}{2\sqrt{x}}$$

SUBSTITUTING $x=4$

$$\left.\frac{dy}{dx}\right|_{x=4} = -\frac{8}{4^2} + \frac{3}{2\sqrt{4}}$$

$$= -\frac{8}{16} + \frac{3}{4}$$

$$= -\frac{1}{2} + \frac{3}{4}$$

$$= \frac{1}{4}$$

IYGB - MP1 PAPER K - QUESTION 2

a) COMPLETING THE SQUARE

$f(x) = 4x^2+4x-1$

$f(x) = 4\left[x^2+x-\frac{1}{4}\right]$

$f(x) = 4\left[\left(x+\frac{1}{2}\right)^2-\frac{1}{4}-\frac{1}{4}\right]$

$f(x) = 4\left(x+\frac{1}{2}\right)^2-1-1$

$f(x) = 4\left(x+\frac{1}{2}\right)^2-2$

ALTERNATIVE

$f(x) = 4x^2+4x-1$

$f(x) = (4x^2+4x+1)-2$

$f(x) = (2x+1)^2-2$

b) SOLVING THE EQUATION USING PART (a)

$f(x)=0$

$4x^2+4x-1=0$

$4\left(x+\frac{1}{2}\right)^2-2=0$

$4\left(x+\frac{1}{2}\right)^2=2$

$\left(x+\frac{1}{2}\right)^2=\frac{1}{2}$

$x+\frac{1}{2} = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}$

$x+\frac{1}{2} = \pm\frac{\sqrt{2}}{2}$

$x = -\frac{1}{2}\pm\frac{\sqrt{2}}{2}$

ALTERNATIVE

$f(x)=0$

$(2x+1)^2-2=0$

$(2x+1)^2=2$

$2x+1=\pm\sqrt{2}$

$2x=-1\pm\sqrt{2}$

$x=-\frac{1}{2}\pm\frac{\sqrt{2}}{2}$

IYGB − MP1 PAPER K − QUESTION 3

PROCEED AS FOLLOWS

$$f(x) = x^{\frac{3}{2}} - 8x^{-\frac{1}{2}}$$

$$f(3) = 3^{\frac{3}{2}} - 8 \times 3^{-\frac{1}{2}} = (\sqrt{3})^3 - \frac{8}{\sqrt{3}} = 3\sqrt{3} - \frac{8}{\sqrt{3}}$$

$$= 3\sqrt{3} - \frac{8\sqrt{3}}{\sqrt{3}\sqrt{3}} = 3\sqrt{3} - \frac{8\sqrt{3}}{3} = 3\sqrt{3} - \frac{8}{3}\sqrt{3}$$

$$= \frac{1}{3}\sqrt{3}$$

i.e. $k = \frac{1}{3}$

IYGB – MP1 PAPER K – QUESTION 4

a) EXPANDING USING PASCAL'S TRIANGLE OR CALCULATOR

$$(2x+4)^3 = 1(2x)^3(4)^0 + 3(2x)^2(4)^1 + 3(2x)^1(4)^2 + 1(2x)^0(4)^3$$

$$\begin{array}{cccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \\ x^3 & & x^2 & & x^1 & & x^0 \end{array}$$

$$(2x+4)^3 = 8x^3 + 48x^2 + 96x + 64$$

b) USING PART (a)

$$(2x-1)(2x+4)^3 = (2x-1)(8x^3 + 48x^2 + 96x + 64)$$

$$\begin{aligned} = \;& 16x^4 + 96x^3 + 192x^2 + 128x \\ & \qquad - 8x^3 - 48x^2 - 96x - 64 \\ \hline & 16x^4 + 88x^3 + 144x^2 + 32x - 64 \end{aligned}$$

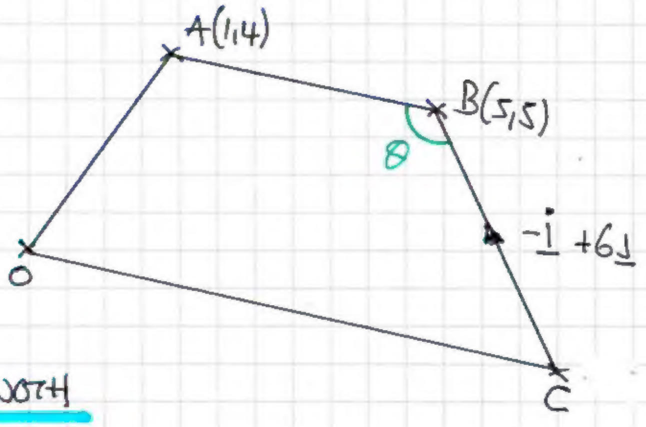
IYGB − MP1 PAPER K − QUESTION 5

a) START WITH A DIAGRAM

$\vec{OA} = \underline{i} + 4\underline{j}$

$\vec{OB} = 5\underline{i} + 5\underline{j}$

$\vec{CB} = -\underline{i} + 6\underline{j}$



FIND $\vec{AB}$, FOLLOWED BY ITS LENGTH

$$\vec{AB} = \vec{AO} + \vec{OB} = -(\underline{i}+4\underline{j}) + (5\underline{i}+5\underline{j}) = 4\underline{i} + \underline{j}$$

$$|\vec{AB}| = |4\underline{i}+\underline{j}| = \sqrt{4^2+1^2} = \sqrt{17}$$

b) NEXT FIND THE VECTOR $\vec{AC}$

$$\vec{AC} = \vec{AB} + \vec{BC} = \vec{AB} - \vec{CB} = (4\underline{i}+\underline{j}) - (-\underline{i}+6\underline{j}) = 5\underline{i} - 5\underline{j}$$

NEXT THE MODULI OF $\vec{AC}$

$$|\vec{AC}| = |5\underline{i} - 5\underline{j}| = \sqrt{5^2+(-5)^2} = \sqrt{25+25} = \sqrt{50}$$

c) FINALLY THE LENGTH OF CB

$$|\vec{CB}| = |-\underline{i}+6\underline{j}| = \sqrt{(-1)^2+6^2} = \sqrt{1+36} = \sqrt{37}$$

BY THE COSINE RULE

$$\Rightarrow |AC|^2 = |AB|^2 + |CB|^2 - 2|AB||CB|\cos\theta$$

$$\Rightarrow (\sqrt{50})^2 = (\sqrt{17})^2 + (\sqrt{37})^2 - 2\sqrt{17}\sqrt{37}\cos\theta$$

$$\Rightarrow 50 = 17 + 37 - 2\sqrt{629}\cos\theta$$

$$\Rightarrow 2\sqrt{629}\cos\theta = 4$$

$$\Rightarrow \cos\theta = 0.079745\ldots$$

$$\therefore \theta = 85.4°$$

IYGB - MP1 PAPER K - QUESTION 6

a) LET $f(x) = x^3 + x^2 - 10x + 8$

looking for factors, trying $\pm 1, \pm 2, \pm 4, \pm 8$

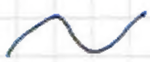
$f(1) = 1 + 1 - 10 + 8 = 0$

$\therefore (x-1)$ IS A FACTOR OF $f(x)$

BY LONG DIVISION OR MANIPULATIONS

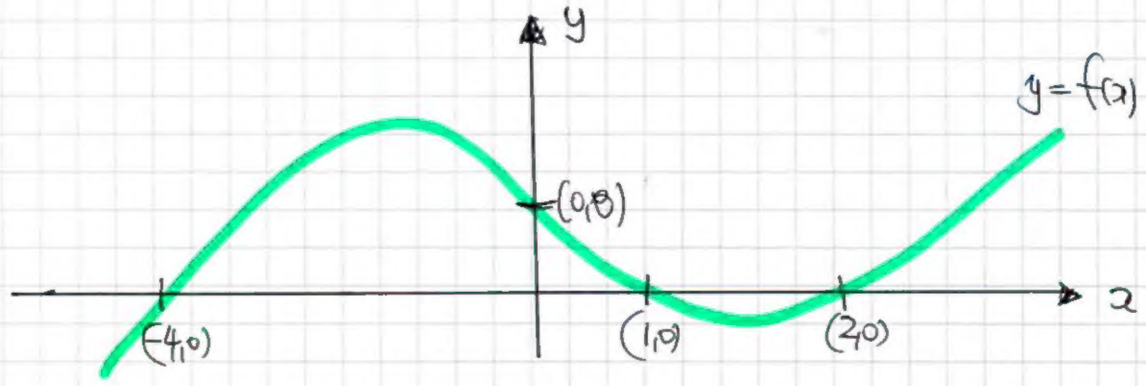
$$f(x) = x^3 + x^2 - 10x + 8 = x^2(x-1) + 2x(x-1) - 8(x-1)$$
$$= (x-1)(x^2+2x-8)$$
$$= (x-1)(x-2)(x+4)$$

b) COLLECTING ALL INFORMATION

$+x^3 \Rightarrow$ 

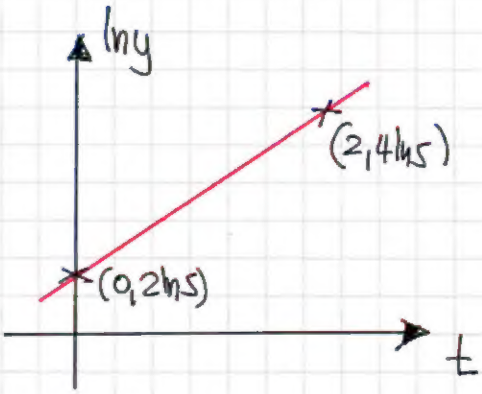
$x=0,\ y=8 \Rightarrow (0,8)$

$y=0,\ x = 1,\ 2,\ -4 \Rightarrow (1,0),\ (2,0),\ (-4,0)$



IYGB - MP1 PAPER K - QUESTION 7

PROCEED AS FOLLOWS



$$\text{GRADIENT} = \frac{4\ln 5 - 2\ln 5}{2 - 0}$$

$$= \frac{2\ln 5}{2}$$

$$= \ln 5$$

THE EQUATION OF THE STRAIGHT LINE IS

$$\Rightarrow \ln y - 2\ln 5 = (\ln 5)(t - 0)$$

$$\Rightarrow \ln y - 2\ln 5 = t\ln 5$$

$$\Rightarrow \ln y = t\ln 5 + 2\ln 5$$

$$\Rightarrow \ln y = \ln 5^t + \ln 5^2$$

$$\Rightarrow \ln y = \ln(5^t \times 5^2)$$

$$\Rightarrow \ln y = \ln(5^{t+2})$$

$$\Rightarrow y = 5^{t+2}$$

IYGB - MP1 PAPER K - QUESTION 8

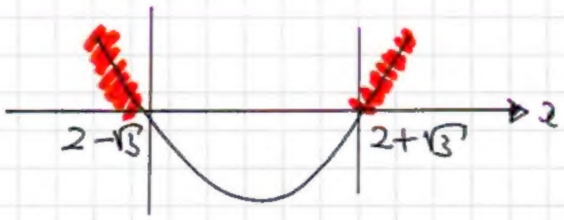
SOLVING THE FIRST INEQUALITY

$$\Rightarrow (x+2)(x+4) > 10x+7$$
$$\Rightarrow x^2+6x+8 > 10x+7$$
$$\Rightarrow x^2-4x+1 > 0$$

THIS DOES NOT FACTORIZE NICELY — OBTAIN CRITICAL VALUES BY COMPLETING THE SQUARE IN THE CORRESPONDING EQUATION — OR USE QUADRATIC FORMULA

$$\Rightarrow \text{"}x^2-4x+1=0\text{"}$$
$$\Rightarrow (x-2)^2-4+1=0$$
$$\Rightarrow (x-2)^2=3$$
$$\Rightarrow x-2=\pm\sqrt{3}$$
$$\Rightarrow x=2\pm\sqrt{3}$$

LOOKING AT THE DIAGRAM



$x<2-\sqrt{3}$ OR $x>2+\sqrt{3}$

SOLVING THE SECOND INEQUALITY

$$\Rightarrow x\sqrt{3} < 2+\frac{2(2x-1)}{\sqrt{3}}$$
$$\Rightarrow 3x < 2\sqrt{3}+2(2x-1)$$
$$\Rightarrow 3x < 2\sqrt{3}+4x-2$$

IYGB - MP1 PAPER K - QUESTION 8

$\Rightarrow \quad -x < -2 + 2\sqrt{3}$

$\Rightarrow \quad x > 2 - 2\sqrt{3}$

MULTIPLIED BY NEGATIVE

FINALLY TO OBTAIN THE COMMON SOLUTION INTERVAL

$\therefore \quad 2-2\sqrt{3} < x < 2-\sqrt{3} \quad \text{OR} \quad x > 2+\sqrt{3}$

IYGB - MP1 PAPER K - QUESTION 9

OBTAIN THE CO-ORDINATES OF A & B

$$\left.\begin{array}{l} y = 10 - 4x - x^2 \\ y = 6 - x \end{array}\right\} \Rightarrow 6 - x = 10 - 4x - x^2$$

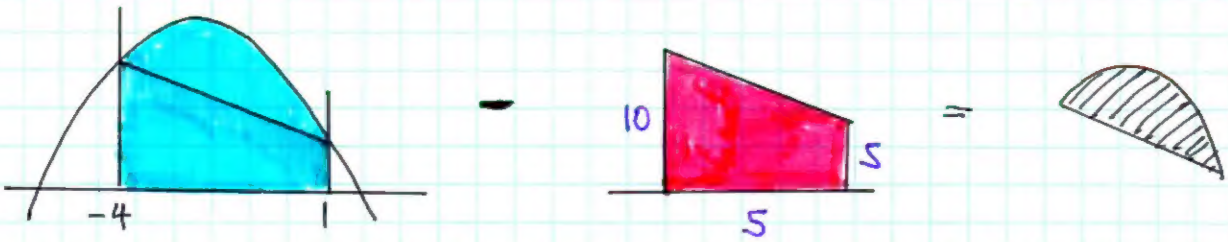
$$\Rightarrow x^2 + 3x - 4 = 0$$

$$\Rightarrow (x+4)(x-1) = 0$$

$$\Rightarrow x = \left\langle \begin{array}{l} -4 \\ 1 \end{array} \right. \quad y = \left\langle \begin{array}{l} 10 \\ 5 \end{array} \right.$$

$$\therefore \; A(-4,10) \quad B(1,5)$$

LOOKING AT THE PICTORIAL EQUATION BELOW



$$\int_{-4}^{1} 10 - 4x - x^2 \, dx$$

$$= \left[10x - 2x^2 - \tfrac{1}{3}x^3\right]_{-4}^{1}$$

$$= \left(10 - 2 - \tfrac{1}{3}\right) - \left(-40 - 32 + \tfrac{64}{3}\right)$$

$$= \frac{23}{3} - \left(-\frac{152}{3}\right)$$

$$= \frac{175}{3}$$

$$\tfrac{1}{2}(10+5) \times 5 = \frac{75}{2}$$

REQUIRED AREA $= \dfrac{175}{3} - \dfrac{75}{2} = \dfrac{125}{6}$

IYGB – MP1 PAPER K – QUESTION 10

SOLVING SIMULTANEOUSLY

$$y = 9^x \quad \& \quad y = 6 \times 5^x$$

$$9^x = 6 \times 5^x$$

TAKING LOGARITHMS BASE 3

$$\Rightarrow \log_3 9^x = \log_3 (6 \times 5^x)$$

$$\Rightarrow x \log_3 9 = \log_3 6 + \log_3 5^x$$

$$\Rightarrow x \log_3 9 = \log_3 6 + x \log_3 5$$

$$\Rightarrow x \log_3 9 - x \log_3 5 = \log_3 6$$

$$\Rightarrow x \left[\log_3 9 - \log_3 5 \right] = \log_3 6$$

$$\Rightarrow x = \frac{\log_3 6}{\log_3 9 - \log_3 5}$$

TIDY FURTHER AS FOLLOWS

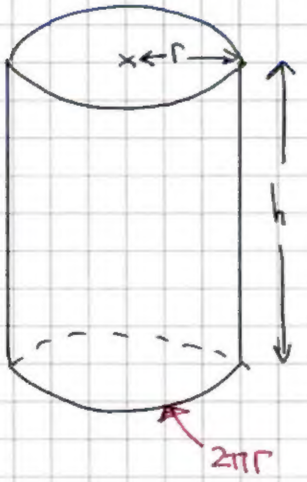
$$\Rightarrow x = \frac{\log_3 3 + \log_3 2}{\log_3 3^2 - \log_3 5}$$

$$\Rightarrow x = \frac{1 + \log_3 2}{2\log_3 3 - \log_3 5}$$

$$\Rightarrow x = \frac{1 + \log_3 2}{2 - \log_3 5}$$

IYGB - MP1 PAPER K - QUESTION 11

a)



CONSTRAINT SURFACE AREA $= 192\pi$

$\Rightarrow$  $= 192\pi$

$\Rightarrow 2\cancel{\pi}rh + 2\times\cancel{\pi}r^2 = 192\cancel{\pi}$

$\Rightarrow rh + r^2 = 96$

$\boxed{rh = 96 - r^2}$

VOLUME $= \pi r^2 \times h$

$\Rightarrow V = \pi r(rh)$

$\Rightarrow V = \pi r(96 - r^2)$

$\Rightarrow V = 96\pi r - \pi r^3$ // AS REQUIRED

b) DIFFERENTIATE & SOLVE FOR ZERO

$\Rightarrow V = 96\pi r - \pi r^3$

$\Rightarrow \frac{dV}{dr} = 96\pi - 3\pi r^2$

$\Rightarrow 0 = 96\pi - 3\pi r^2$

$\Rightarrow 3\cancel{\pi}r^2 = 96\cancel{\pi}$

$\Rightarrow r^2 = 32$

$\Rightarrow r = +\sqrt{32} \approx 5.66$ cm //

IYGB - MP1 PAPER K - QUESTION 11

c) CHECKING WITH THE 2ND DERIVATIVE

$$\Rightarrow \frac{dV}{dr} = 96\pi - 3\pi r^2$$

$$\Rightarrow \frac{d^2V}{dr^2} = -6\pi r$$

$$\Rightarrow \left.\frac{d^2V}{dr^2}\right|_{r=5.66} = -106.62\ldots < 0$$

INDEED IT WILL GIVE THE MAXIMUM VALUE FOR V

d) $V = 96\pi r - \pi r^3$

$$\Rightarrow V_{MAX} = 96\pi(4\sqrt{2}) - \pi(4\sqrt{2})^3$$

$$\Rightarrow V_{MAX} = 4\pi\sqrt{2}\left[96 - (4\sqrt{2})^2\right]$$

$$\Rightarrow V_{MAX} = 4\pi\sqrt{2} \times 64$$

$$\Rightarrow V_{MAX} = 256\pi\sqrt{2}$$

$$\Rightarrow V_{MAX} \simeq 1137$$

IYGB - MP1 PAPER K - QUESTION 12

a) PROCEED AS FOLLOWS

- GRADIENT $AB = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-1}{6-0} = \frac{4}{6} = \frac{2}{3}$

- OUR LINE (PERPENDICULAR BISECTOR) $= -\frac{3}{2}$

- FINDING THE MIDPOINT OF AB : $\left(\frac{0+6}{2}, \frac{1+5}{2}\right) = (3,3)$

- REQUIRED LINE HAS EQUATION

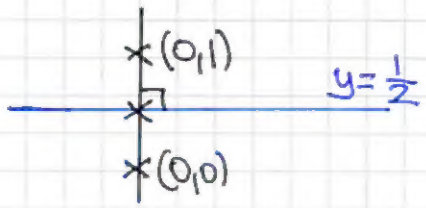
$$y - y_0 = m(x - x_0)$$
$$y - 3 = -\frac{3}{2}(x-3)$$
$$2y - 6 = -3x + 9$$
$$2y + 3x = 15$$

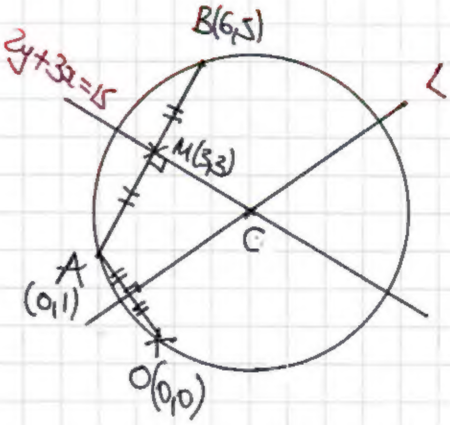
b) LOOKING AT A DIAGRAM – NOT DRAWN TO SCALE

GRADIENT $OA = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{0-0} = \frac{1}{0}$

I.E INFINITE GRADIENT

IE LINE IS VERTICAL





$\therefore$ PERPENDICULAR BISECTOR HAS EQUATION $y = \frac{1}{2}$

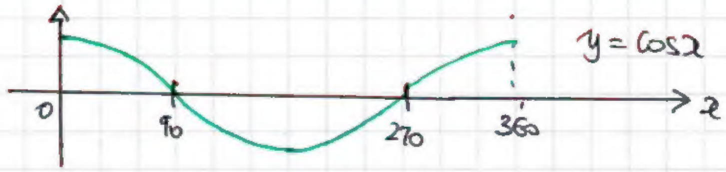
USING $2y + 3x = 15$ WITH $y = \frac{1}{2}$

$$2 \times \frac{1}{2} + 3x = 15$$
$$3x = 14$$
$$x = \frac{14}{3}$$

$\therefore C\left(\frac{14}{3}, \frac{1}{2}\right)$

IYGB – MP1 PAPER K – QUESTION 13

a) LOOKING AT A SKETCH OF THE GRAPH OF $y = \cos x$



TRANSLATING BY 60° TO THE "LEFT"

$\therefore \quad (270, 0) \longmapsto B(210, 0)$

$\qquad (360, 1) \longmapsto C(300, 0)$

& SUBSTITUTING $x = 0$ INTO $y = \cos(x + 60)$ GIVES $y = \frac{1}{2}$

$\therefore \quad A(0, \frac{1}{2})$

b) SOLVING THE EQUATION $y = -\frac{1}{2}$

$\Longrightarrow \cos(x + 60) = -\frac{1}{2}$

$\arccos(-\frac{1}{2}) = 120°$

$\Longrightarrow \begin{cases} x + 60° = 120° \pm 360n \\ x + 60° = 240° \pm 360n \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\Longrightarrow \begin{cases} x = 60° \pm 360n \\ x = 180° \pm 360n \end{cases}$

$\therefore \quad x_1 = 60°$

$\qquad x_2 = 180°$

IE $P(60°, -\frac{1}{2})$ & $Q(180, -\frac{1}{2})$